Pattern Recognition Exercises
Sheet 1 “Bayes Classification”

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Submission Form:	Email to **chen.li@uni-siegen.de** with a PDF attachment
File Name:	PR-ES1-Surname.pdf
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1 Bayes Decision Theory – Fundamental Terms (6 Points)

Please explain the following terms associated with the Bayes classification theory, namely the *a priori probability*, the *a posteriori probability*, and the *likelihood density function* for a particular class ω and a particular pattern $\boldsymbol{x}$ to be classified! Please do it by giving an example of a statistical classification task! Please describe the example dividing it into the the training and the classification phase!

2 Bayes Decision Theory (6 Points)

The probability for the occurrence of an emoticon in a particular online chat (event A) amounts to $P(A) = 0.005$. A method for automatic detection of emoticons in chats has been developed. B denotes the event that the method has detected an emoticon. The probability $P(B|A)$ (method has detected an emoticon, if an emoticon was present in a chat) is 99.9%, while the probability $P(B|\bar{A})$ (method has detected an emoticon, although no emoticon was present) amounts to 0.1%.
How is then the probability that an emoticon really occurs in a chat, if the method has returned a positive detection result?

3 Classifiers Based on Bayes Decision Theory (6 Points)

In a two-class problem with a single feature x the pdfs are Gaussian with variance $\sigma^2 = \frac{1}{4}$ for both classes and mean values 0 and 2.

If $P(\omega_1) = P(\omega_2) = \frac{1}{2}$, compute the threshold value x_0
(a) for minimum error probability and

(b) for minimum risk, with $\lambda_{12} = 0.5$ and $\lambda_{21} = 1.0$

4 The Bayesian Classifier for Normally Distributed Classes (6P)

In a three-class, two-dimensional problem the feature vectors in each class are normally distributed with the following covariance matrix:

$$\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \boldsymbol{\Sigma}_3 = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix}$$

The mean vectors for the classes are $\boldsymbol{\mu}_1 = [0.1, 0.1]^{\mathrm{T}}$, $\boldsymbol{\mu}_2 = [2.1, 1.9]^{\mathrm{T}}$, $\boldsymbol{\mu}_3 = [-1.5, 2.0]^{\mathrm{T}}$. Assuming that the classes are equiprobable ($P(\omega_1) = P(\omega_2) = P(\omega_3)$), please classify the feature vector $\boldsymbol{x} = [1.4, 1.2]^{\mathrm{T}}$ according to the Bayes minimum error probability classifier!

5 Maximum Likelihood Parameter Estimation (6P)

The normally distributed training data of a class ω consists of N training samples described by one-dimensional features $x_1, x_2, \ldots, x_N$. The standard deviation for the data is known and amounts to σ. Please estimate the the mean value μ for the training data using the Maximum Likelihood optimisation! Please use the so called log-likelihood function for this!