Pattern Recognition Exercises
Sheet 2 "Non-Bayes Classification"

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File Name:	PR-ES2-Surname.pdf
Exercise Discussion:	May 20, 2014, 8:30am, H-F 001

1 Linear Classifiers - The Perceptron Algorithm (8 Points)

Consider a two-class problem with two-dimensional feature vectors $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$ distributed in each of the classes ω_1 and ω_2 in the following way

$$p(\boldsymbol{x}|\omega_1) = \frac{1}{\left(\sqrt{2\pi\sigma_1^2}\right)^2} \exp\left(-\frac{1}{2\sigma_1^2}(\boldsymbol{x}-\boldsymbol{\mu_1})^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{\mu_1})\right)$$

$$p(\boldsymbol{x}|\omega_2) = \frac{1}{\left(\sqrt{2\pi\sigma_2^2}\right)^2} \exp\left(-\frac{1}{2\sigma_2^2}(\boldsymbol{x}-\boldsymbol{\mu_2})^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{\mu_2})\right)$$

with

$$\boldsymbol{\mu}_1^{\mathrm{T}} = [1,1], \qquad \boldsymbol{\mu}_2^{\mathrm{T}} = [0,0], \qquad \sigma_1^2 = \sigma_2^2 = 0.2 \qquad .$$

Produce four feature vectors for each class (e. g., by coding a short script for this). To guarantee linear separability of the classes, disregard vectors with $x_1 + x_2 < 1$ for ω_1 and vectors with $x_1 + x_2 > 1$ for ω_2. Use these vectors to design a linear classifier using the perceptron algorithm, whereas $\rho = 0.7$ for all iterations.

2 Linear Classifiers - Sum of Error Squares Estimation (6P)

Consider again the problem of Task 1, but this time three feature vectors of each class without any assumptions[1] have to be produced. Design a classifier for this problem using the sum of error squares criterion.

[1] The classes do not have to be linearly separable.

3 Linear Classifiers - Support Vector Machines (8P)

Design a two-class SVM classifier for the following training features $\omega_1 \rightarrow \{[0,1]^\mathrm{T}, [1,0]^\mathrm{T}\}$ and $\omega_2 \rightarrow \{[-1,0]^\mathrm{T}, [0,-1]^\mathrm{T}\}$ using the KKT conditions.

4 Nonlinear Classifiers - The Backpropagation Algorithm (8P)

Consider a simple neural network having two layers with three nodes in the hidden and one node in the output layer. Perform two iterations of the backpropagation algorithm updating the network weights for a single training pair ($\boldsymbol{x} = [1,1,1]^\mathrm{T}, y = 0.5$).